A model for the COVID-19 evaluated in the middle of 2020

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Six months of COVID-19 into 2020 is the opportunity to have a global look at the situation worldwide.

All kinds of comparisons have been made using different indicators but the use of statistical models to assist in those comparisons is seldom done. This is probably because simple global statistical indicators are not fitted to actual data.

In this document, we propose a simple equation to model the dynamics of the number of cases and the number of deaths, and we use the coefficients of the model to compare the different countries in their initial infection rates and in their reaction to the infection.

The model was fitted to weekly averages of daily new cases and deaths provided by the European Centre for Disease Prevention and Control on the geographic distribution of COVID-19 cases worldwide (https://www.ecdc.europa.eu/en/publications-data/download-todays-data-geographic-distribution-covid-19-cases-worldwide).

The model proposed

Models of population growth exist at least since the 1798 publication by Malthus of "An Essay on the Principle of Population", with the well-known equation of exponential growth that can be simply expressed as:

$N(t) = N(0)\, e^{r t}$, or $N(t) = N(0)\, (1 + \alpha)^{t}$

Where

$N(t)$ is the population number at time t, $N(0)$ is the population number at time 0, and r and α are growth rate parameters, with $r = \ln (1 + \alpha)$.

If we start when $N(0)=1$ and we make $a = 1 + \alpha$, we can simply rewrite the equation of exponential growth as:

$$N(t) = a^{t}$$

The recognition that growth cannot continue indefinitely was included in population models by Benjamin Gompertz who, in his publication in 1825, the "Law of human mortality" proposed functions setting a limit of growth as the maximum possible size of the population. In the same sequence, Verhulst published in 1838, his "Notice sur la loi que la population poursuit dans son accroissement" which presented the first mathematical formulation of what we call the logistic growth model. In the logistic model, the effect of the logistic growth rate (k) is limited by a maximum population size (L):

$$N(t) = L / [1 + e^{-k (t - ts)}]$$

The value ts is the midpoint of the resulting sigmoid curve. Growth rate decreases with increasing t and as N approaches its theoretical limit, L.

The logistic equation has been widely used and extended. In 1959, Richards proposed a flexible approach for the logistic growth function, which was further extended to what is known as a generalized Richards model. This model has been used in fitting a variety of logistic-type epidemic curves (Chowell 2017). In 2020, Wu et al. (2020) used this approach for the analysis of data of the COVID-19 outbreak in China and other parts of the world.

Here we propose a different model to represent an episode of population growth and reaction to growth, with an application to the outbreaks of the COVID-19, but with general use.

The proposed equation has the form:

$$N(t) = a^{t\, b^{\ t}}$$

where:

a is an infection rate, similar to the growth rate previously mentioned, and

b is a reaction rate.

If b = 1 there is no reaction to the infection and the equation is the simple exponential function. As b departs from unity, the infection becomes gradually more distinct from the exponential form and there is a gradual decrease in population numbers.

If we want to start with N(0)=1 we have to define a time t=i as that of the first case. With this we have:

$$N(t) = a^{(t-i)\, b^{\ (t-i)}}$$

This was the basic simple equation used in this modelling exercise for the number of cases and deaths for various countries worldwide in this first semester of 2020.

However, in some cases the data showed clearly two episodes. In this case a sum of two equations was fitted, one episode starting at day i and the other at day j. In some cases i=j as for the models of deaths with two episodes. In the case of two episodes the full equation used was of the form:

$$N(t) = (a1)^{(t-i)\,(b1\,)^{(t-i)}} + (a2)^{(t-j)\,(b2\,)^{(t-j)}}$$

Where:

N is the number of cases or the number of deaths

a1 and a2 are the initial infection rates of the two simultaneous episodes,

b1 and b2 are reaction rates in the two episodes (1 represents no reaction),

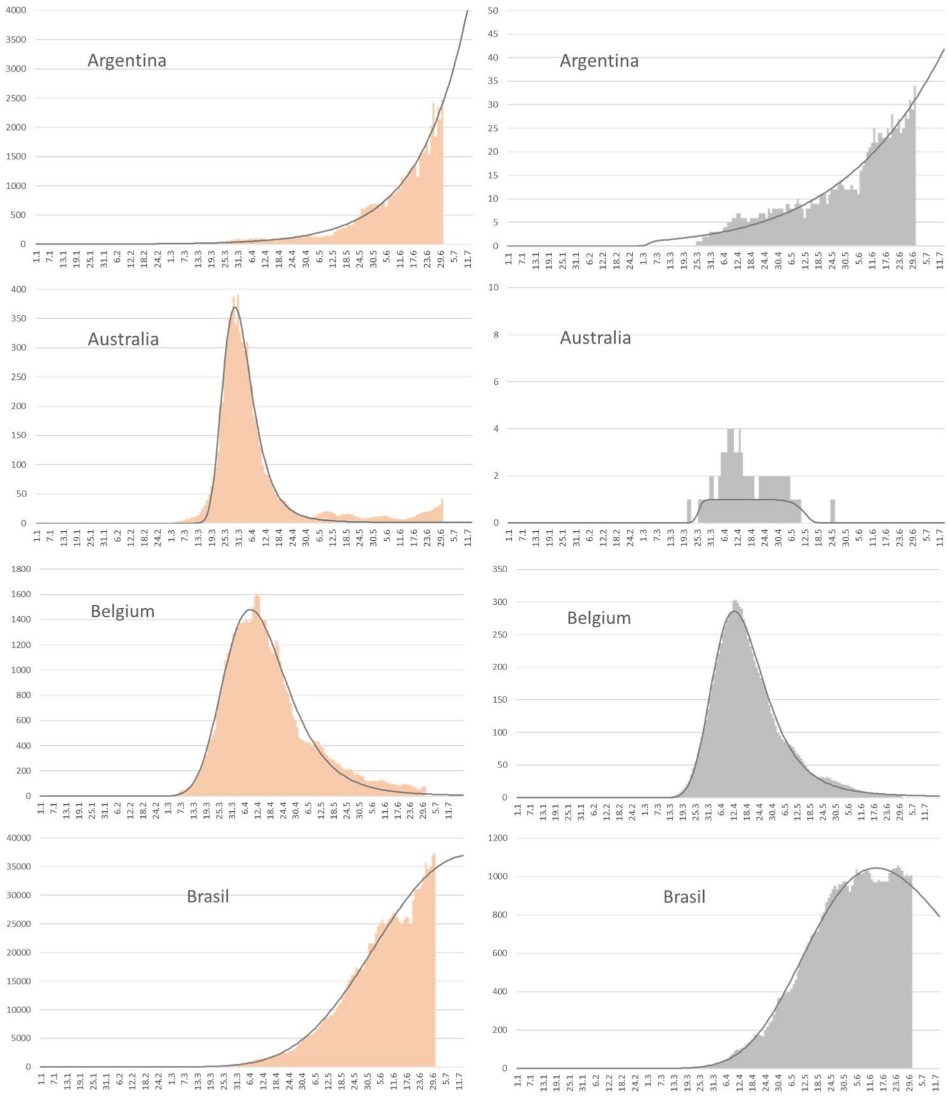
t is time (in days from January 1st 2020)

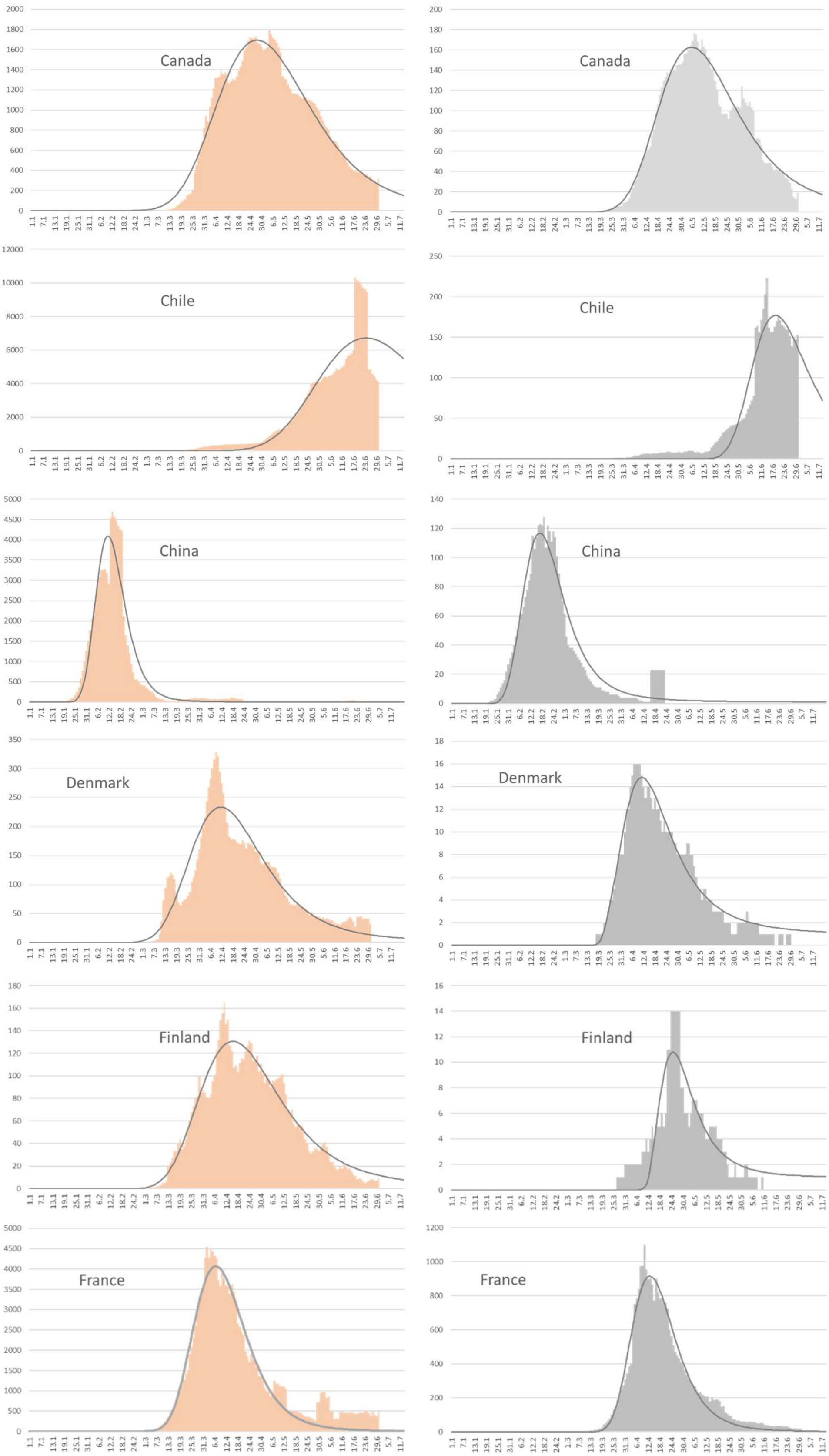
i and j are the days of the start of the two episodes for the cases or the deaths (from January 1st 2020)

The results

The results of the two models are presented in the graphs. The estimated values for the coefficients a1, b1, a2, b2, i, j, and R^2 are presented for various countries in the Annex.

The graphs correspond to various countries, in alphabetic order, representing the weekly averages of the number of cases (left) and deaths (right), in solid bars, and the fitted equations in lines:

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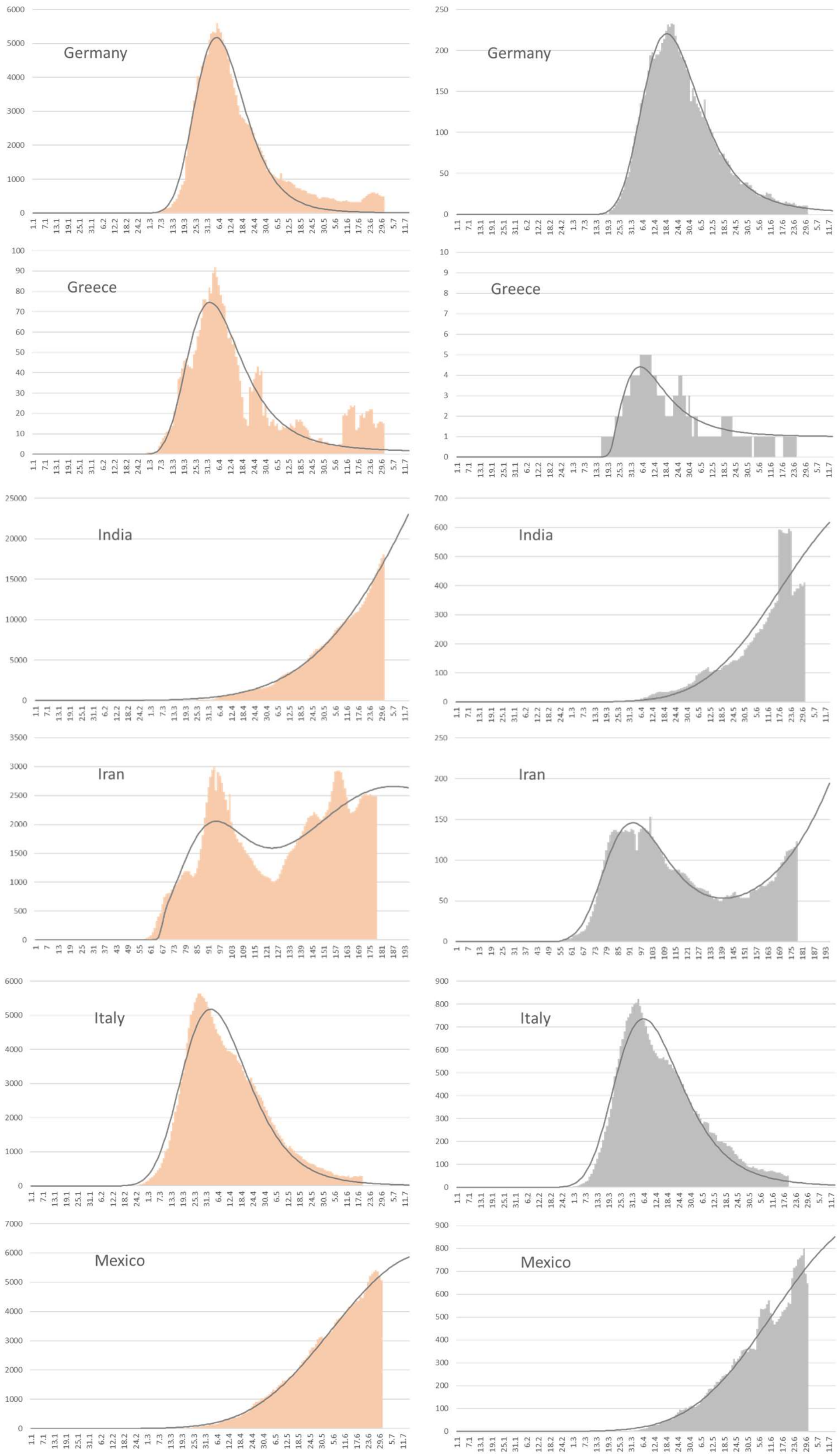
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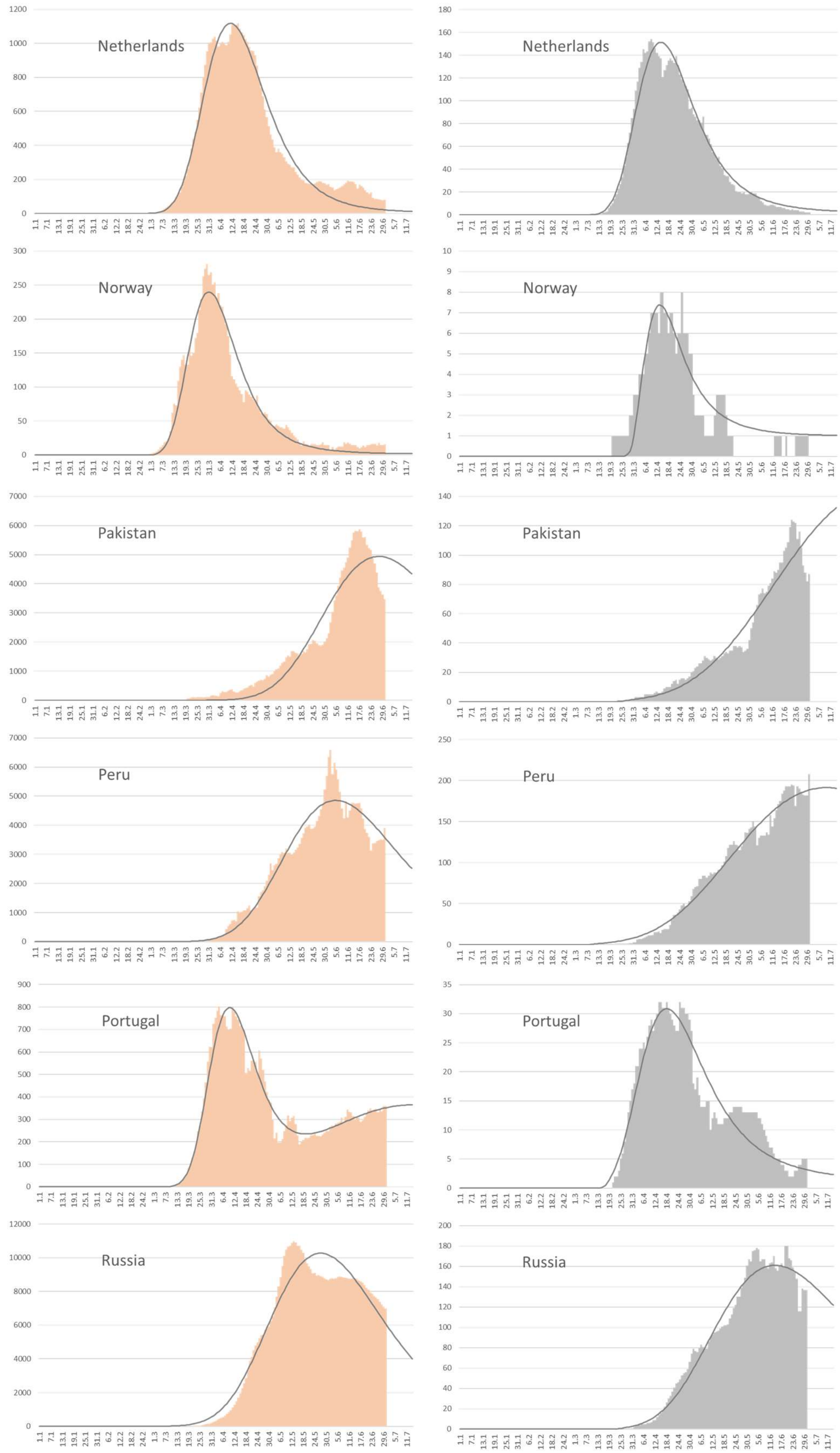
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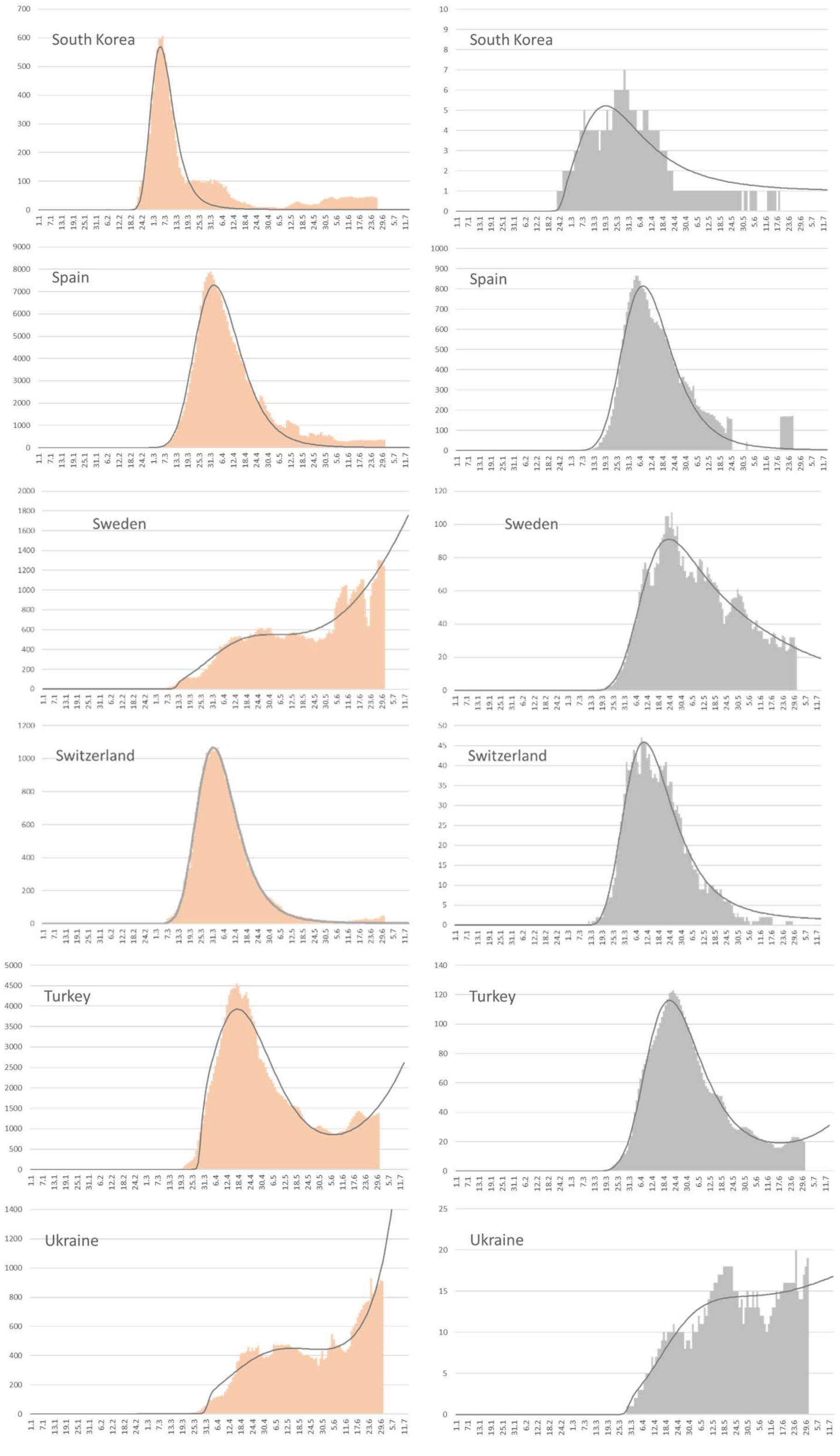
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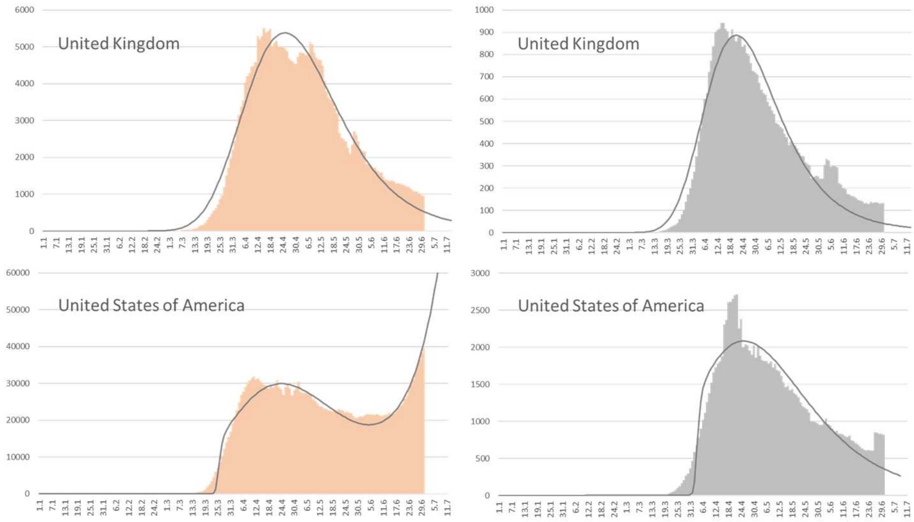

Figure 1. Results for various countries showing the weekly average of the observed number of cases in solid bars and the fitted equations (left), and the weekly average of deaths in solid bars and the corresponding fitted equations (right).

There is a general very good agreement between observed values and fitted equations showing the adequacy of the model to represent the episodes of COVID-19 in various countries.

A careful interpretation of the results reveals differences in infection dates, and in the infection and reaction rates, allowing for simulations with different scenarios. In Figure 2 we can see that the start of the cases and the start of the deaths was different in the different countries from the early episodes in China to the latest episodes in Pakistan or Chile.

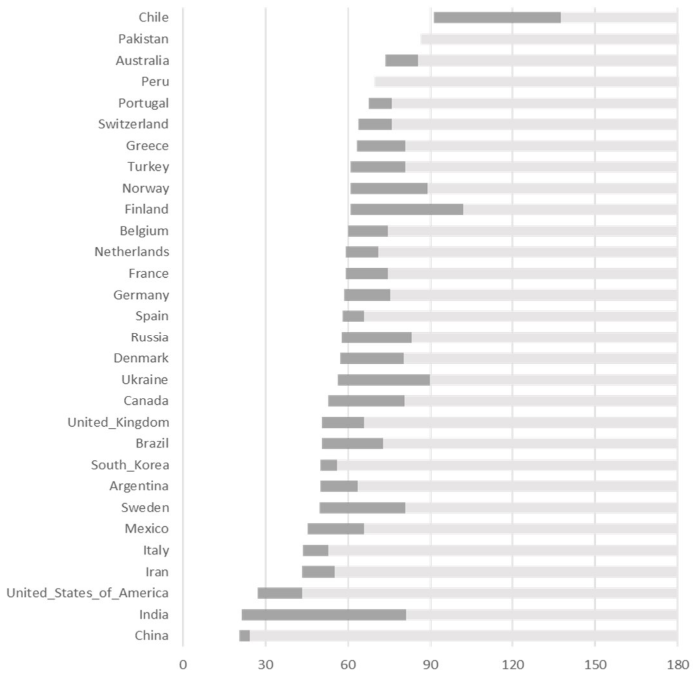

Figure 2. Estimated dates of the start of cases and deaths in different countries. The darker portions of the bars correspond to the period between the start of cases and the start of deaths.

The analysis of the coefficients (Figure 3) also allows for interesting comparisons.

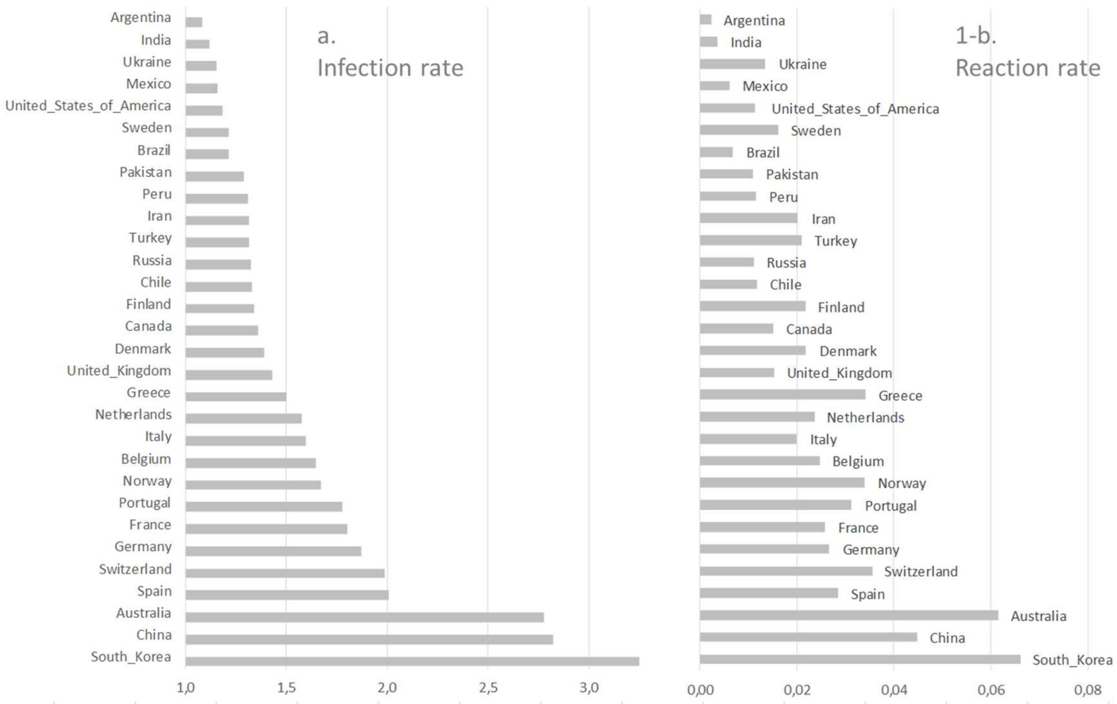

Figure 3. The coefficients for the infection rate (a) and reaction rate (1-b) organized by order of the infection rate.

From Figure 3 it is clear that the two coefficients are correlated. In some countries (South Korea, China and Australia) the infections rate are higher and the corresponding reaction rates are also higher, resulting in fast and strong responses and shorter duration episodes. In the other extreme, Argentina, India, Ukraine, Mexico, United States of America, Sweden and Brasil show lower infection rates but also very small reaction rates, resulting in longer episodes with higher number of cases.

For the number of deaths a similar analysis was done and the results are presented in Figure 4. Fast increases in the number of deaths in countries as in France, Spain, or China corresponded to rapid reactions. On the other hand countries as Australia, Argentina, Pakistan, Peru, Mexico, India, or the United States of America show slower initial mortality rates but also slower response rates.

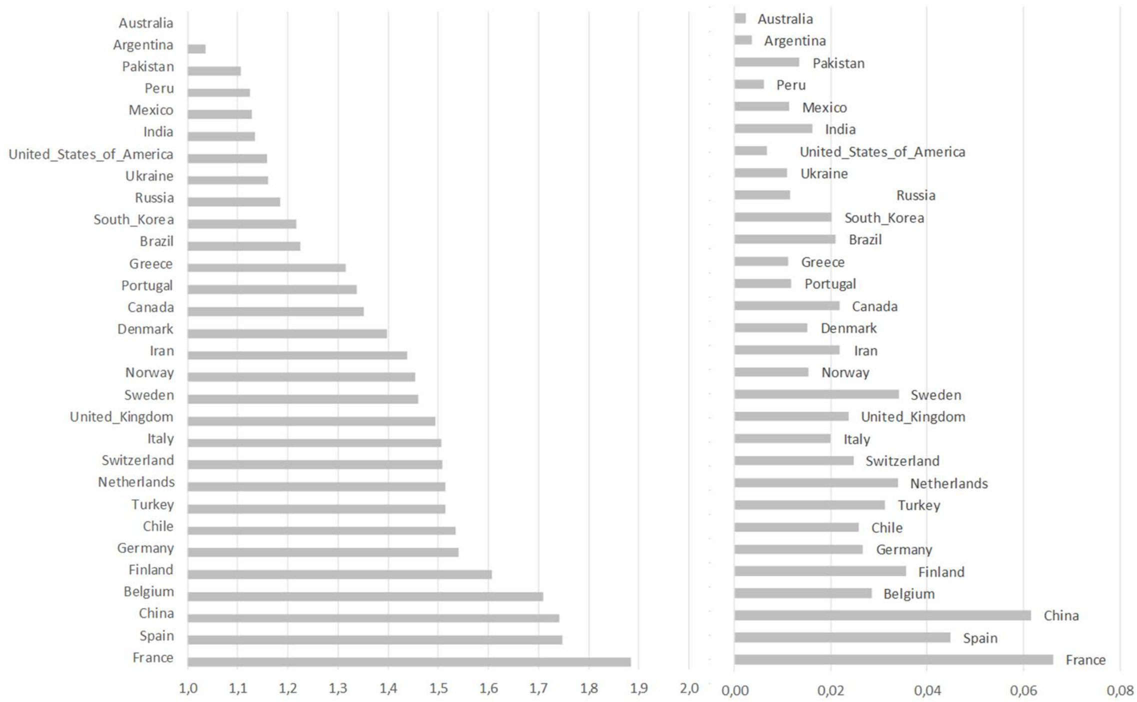

Figure 4. Coefficients for the models of deaths. The mortality growth rate (left) and the reaction rate to that mortality (right) show very significant differences between countries.

Finally, it is concluded that the model proposed was very useful to distinguish between different regions of a country, as in Portugal, or the various States of the United States of America. This will be presented in a later phase.

Annex: Results

Table 1. Coefficients of the equation for the model of cases

	Parameters for the equation of Number of Cases						
Countries	a1	b1	i	a2	b2	j	R^2
Argentina	1,0827	0,9977	50,0				0,984
Australia	2,7772	0,9384	73,7				0,983
Belgium	1,6456	0,9752	60,0				0,987
Brazil	1,2140	0,9932	50,7				0,992
Canada	1,3602	0,9849	52,9				0,981
Chile	1,3285	0,9882	91,2				0,891
China	2,8194	0,9552	20,7				0,924
Denmark	1,3885	0,9781	57,3				0,903
Finland	1,3374	0,9783	60,9				0,964
France	1,8034	0,9742	59,2				0,951
Germany	1,8736	0,9734	58,6				0,971
Greece	1,5010	0,9659	63,4				0,883
India	1,1158	0,9963	21,5				0,997
Iran	1,3122	0,9798	43,4	1,1362	0,9931	64,5	0,917
Italy	1,5968	0,9801	43,6				0,975
Mexico	1,1563	0,9939	45,5				0,998
Netherlands	1,5770	0,9764	59,2				0,974
Norway	1,6722	0,9661	61,0				0,963
Pakistan	1,2871	0,9891	86,5				0,940
Peru	1,3075	0,9884	69,7				0,971
Portugal	1,7749	0,9688	67,7	1,1327	0,9922	67,7	0,979
Russia	1,3250	0,9889	57,8				0,975
South_Korea	3,2504	0,9339	50,0				0,927
Spain	2,0067	0,9716	58,1				0,976
Sweden	1,2122	0,9839	49,7	1,0846	0,9963	70,6	0,957
Switzerland	1,9849	0,9645	63,8				0,997
Turkey	1,3159	0,9791	61,1	1,0774	0,9978	87,1	0,974
Ukraine	1,1508	0,9866	56,3	1,0405	1,0033	90,1	0,899
United_Kingdom	1,4308	0,9848	50,8				0,976
nited_States_of_America	1,1844	0,9886	27,1	1,0702	1,0000	82,9	0,984

Table 2. Coefficients of the equation for model of deaths

Countries	a1	b1	i	a2	b2	R^2
Argentina	1,0358	0,9984	63,6			0,970
Australia	1,0000	1,3219	85,5			0,492
Belgium	1,7097	0,9657	74,6			0,996
Brazil	1,2253	0,9893	72,9			0,997
Canada	1,3523	0,9784	80,7			0,979
Chile	1,5351	0,9700	137,6			0,957
China	1,7411	0,9580	24,4			0,401
Denmark	1,3986	0,9552	80,2			0,977
Finland	1,6079	0,9293	102,1			0,843
France	1,8839	0,9664	74,5			0,976
Germany	1,5401	0,9710	75,5			0,995
Greece	1,3160	0,9341	80,9			0,875
India	1,1337	0,9930	81,3			0,917
Iran	1,0461	0,9987	55,3	1,4383	0,9733	0,983
Italy	1,5064	0,9774	53,0			0,976
Mexico	1,1282	0,9935	66,0			0,985
Netherlands	1,5138	0,9701	71,1			0,988
Norway	1,4551	0,9333	89,0			0,869
Pakistan	1,1059	0,9926	85,5			0,962
Peru	1,1250	0,9918	68,8			0,988
Portugal	1,3372	0,9693	76,0			0,948
Russia	1,1853	0,9878	83,3			0,985
South_Korea	1,2173	0,9572	56,2			0,833
Spain	1,7481	0,9698	66,0			0,938
Sweden	1,4602	0,9679	80,9	1,1686	0,9839	0,975
Switzerland	1,5081	0,9613	76,1			0,979
Turkey	1,0207	1,0031	80,9	1,5141	0,9683	0,993
Ukraine	1,1601	0,9760	89,7	1,0428	0,9950	0,956
United_Kingdom	1,4949	0,9784	66,1			0,959
nited_States_of_America	1,1579	0,9863	43,5	1,0034	0,9807	0,949